Lesson 2-5: Proving Angles Congruent

Geometric Proofs

Yesterday we discovered that solving an algebraic expression is essentially doing a proof, provided you justify each step you take. Today we are going to practice doing proofs in geometry by discovering properties of congruent angles using inductive reasoning and then proving they are true using deductive reasoning.

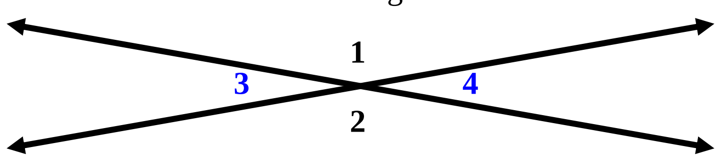
Special angle pairs

We first need to learn about some important angle pairs. They are:

1. Vertical angles
2. Adjacent angles
3. Complementary angles
4. Supplementary angles

Vertical angles

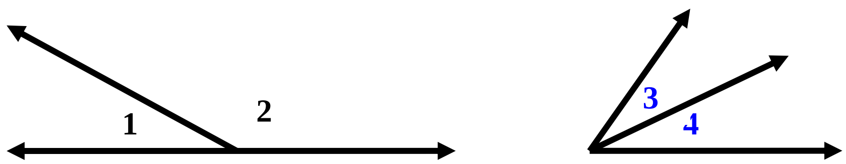
The first is vertical angles. The following picture is an example of vertical angles: $\angle 1$ and $\angle 2$ are vertical angles as are $\angle 3$ and $\angle 4$. Looking at the picture, try to come up with your own definition for vertical angles.



A good working definition is vertical angles are two angles whose sides form two pairs of opposite rays.

Adjacent angles

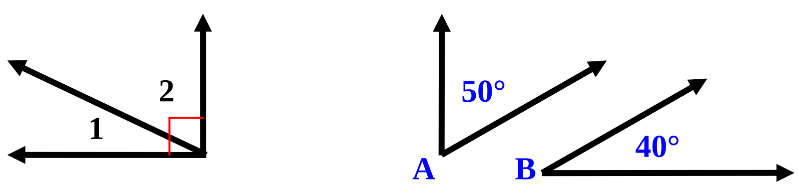
Look at the following picture and develop your own definition for adjacent angles. Angles $\angle 1$ and $\angle 2$ are adjacent angles as are $\angle 3$ and $\angle 4$.



How close were you to something like this? There are three very important properties to note. Adjacent angles are two coplanar angles with: 1) a common side, 2) a common vertex, and 3) no common interior points. Did you get all three?

Complementary angles

Try your hand at the following (angles $\angle 1$ and $\angle 2$ are complementary angles as are $\angle A$ and $\angle B$):



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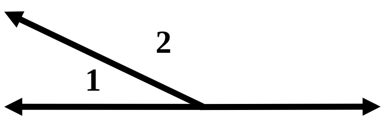
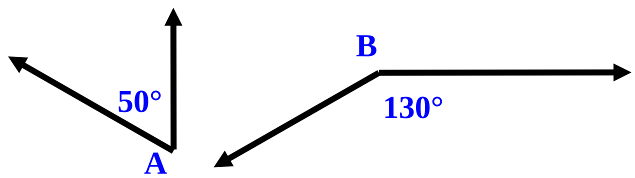
Here we go: complementary angles are two angles whose measures have sum 90. Did you note that they do not have to share a common side or vertex? We can also say "each angle is the *complement* of the other."

Supplementary angles

Last but not least … in the following picture, angles $\angle 1$ and $\angle 2$ are supplementary angles as are $\angle A$ and $\angle B$:





Supplementary angles are two angles whose measures have sum 180. Each angle can be called the *supplement* of the other. Again, did you note they do not have to have a common side or vertex? It is simply based in the sum of their measures.

Take a look at example 1 on page 97 of the text. Try to find all the complementary, supplementary and vertical angles. The answers are provided in the example.

Using diagrams to draw conclusions

A diagram of a geometric figure can provide us with much information allowing us to draw many conclusions. By examining a diagram, we can conclude the following:

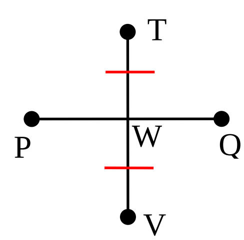
1. adjacent angles
2. adjacent supplementary angles
3. vertical angles

We do need to be careful though; unless the drawing gives you very specific information, you can not conclude any of the following:

1. that angles or segments are congruent (must be marked)
2. an angle is a right angle (must have the ⌐ symbol or indicate the angle is 90°)
3. non-adjacent supplementary angles (must be marked or indicate angle measure)
4. lines are parallel (must be marked – matching arrow head marks mid-line)
5. lines are perpendicular (must be marked with $\perp$ symbol)

Work through example 2 on page 97 of the text. Be careful! It is easy to assume something that you can't! The answers are provided in the example.

Now look Check Understanding 2 on page 97. Given the following diagram, indicate if you can make each of the following conclusions (explain):



a) $\overline{TW} \cong \overline{WV}$

b) $\overline{PW} \cong \overline{WQ}$

c) $\overline{TV} \perp \overline{PQ}$

d) $\overline{TV}$ bisects $\overline{PQ}$

e) W is the midpoint of $\overline{TV}$

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a) Yes; the congruent segments are marked.
b) No; there are no markings.
d) No; there are no markings.
d) No; there are no markings.
e) Yes; the congruent segments are marked.

Applying what we've learned

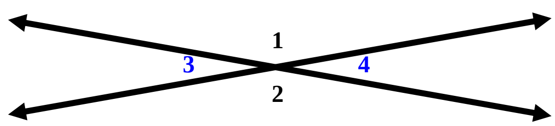
Let's have some fun. Consider the following diagram. From what we've just learned, we can say that angles 1 and 2 are vertical angles as are angles 3 and 4. Examine the picture and form a conjecture about those two angles. If you need to, redraw them on a separate piece of paper and fold the sides of $\angle 1$ onto $\angle 2$ and $\angle 3$ onto $\angle 4$. Based on your observations, what is your conjecture about these angles?

Well, it would be perfectly reasonable to conjecture that vertical angles are congruent. They certainly appear to be … at least these two do. Excellent; we've used inductive reasoning to form a conjecture. Now, let's apply deductive reasoning to prove our conjecture.

Proving vertical angles are congruent

When we do a proof, it is best to state what we know, what our starting point is. It is also good to state what we need to prove (where we need to end up). We do this by stating (for this proof):

Given: $\angle 1$ and $\angle 2$ are vertical angles.
Prove: $\angle 1 \cong \angle 2$



Then, down to business! There are several ways you can write out the proof. You can simply state the steps in paragraph form. Another method is to lay it out just as you do when solving an algebra equation. Either way is acceptable. I prefer the line-by-line method as for me it is easier to follow each step. Here we go…let's prove our conjecture.

First, let's plan out our strategy. Can we get any ideas from the diagram that will help us get going? Well, notice that $\angle 1$ and $\angle 4$ are supplementary. Hey! Keep going around the diagram: $\angle 4$ and $\angle 2$ are also supplementary. We can use this information to come up with an algebraic statement we can manipulate. We start off by stating what we've just realized: name the supplementary angles using the angle addition postulate to show the sum of their measures is 180. We can then, using substitution, show that the sum of the measures of each pair is equal to that of the other. From there on out, it is simple algebra:

Proof:	$m\angle 1 + m\angle 4 = 180$	Angle Addition Postulate
	$m\angle 4 + m\angle 2 = 180$	"" "" ""
	$m\angle 1 + m\angle 4 = m\angle 4 + m\angle 2$	Substitution Property (**all on one side**)
	$m\angle 1 = m\angle 2$	Subtraction Property of = ($-m\angle 4$ **ea side**)
	$\angle 1 \cong \angle 2$	Defn Congruent angles

Q.E.D.! What's that mean? It is a Latin phrase "*quod erat demonstrandum*" meaning "which was demonstrated. We often put it at the end of a proof to show how smart we are.

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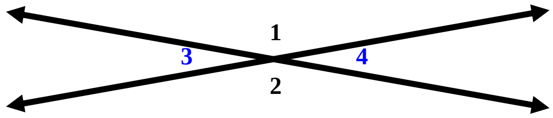
Theorems

Excellent! We have proven our conjecture! If we prove a conjecture, we can call it a <u>theorem</u>. If you recall from yesterday, one of the tools we can use in proofs is "previously accepted or proven geometric conjectures (theorems)." Well, we have just developed our first theorem!

Theorem 2-1 Vertical Angles Theorem

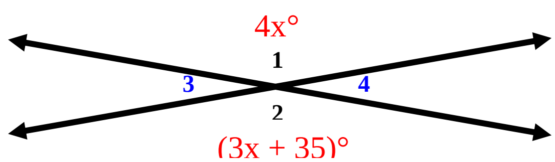
Vertical angles are congruent.

$\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$



Using the Vertical Angles Theorem

This is really quite a powerful little theorem. Consider the following diagram and using the vertical angle theorem, find the measure of each angle. This is example 4 and Check Understanding 4 on page 99 of the text.



Solve for x and justify each step.

Given: $\angle 1 \cong \angle 2$

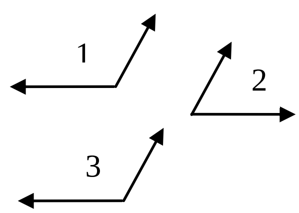
$m\angle 1 = m\angle 2$	Defn congruent angles
4x = 3x + 35	Substitution Property (**all on one side**)
x = 35	Subtraction Property of Equality (**-3x each side**)

$m\angle 1 = m\angle 2 = 140$

$m\angle 3 = m\angle 4 = 180 - 140 = 40$

Try it again

Let's give it one more shot. Consider the following diagram. Given that $\angle 1$ and $\angle 2$ are supplementary angles, and that $\angle 2$ and $\angle 3$ are too, what would you conjecture about $\angle 1$ and $\angle 3$?



If your conjecture is that $\angle 1$ and $\angle 3$ are congruent, that is an excellent conjecture! Now, let's prove it.

Proof

Given: $\angle 1$ and $\angle 2$ are supplementary

$\angle 2$ and $\angle 3$ are supplementary

Prove: $\angle 1 \cong \angle 3$

Proof:	
$m\angle 1 + m\angle 2 = 180$	Angle Addition Postulate
$m\angle 2 + m\angle 3 = 180$	"" "" ""
$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	Substitution
$m\angle 1 = m\angle 3$	Subtraction Prop of =
$\angle 1 \cong \angle 3$	Defn of congruent angles

Since we just proved a conjecture, we now have another theorem!

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Theorem 2-2 Congruent Supplements Theorem

If two angles are supplements of the same angle (or of congruent angles), then the two angles are congruent.

More theorems

Here are the next three theorems. We will leave their proofs for the homework assignment (problems #19, 31, 35, and 56).

Theorem 2-3 Congruent Complements Theorem

If two angles are complements of the same angle (or of congruent angles), then the two angles are congruent.

Theorem 2-4

All right angles are congruent.

Theorem 2-5

If two angles are congruent and supplementary, then each is a right angle.

Assign homework

p. 100 1-25 odd, 29-35, 39-42, 43-53 odd, 56-59